3D Medical Image Processing Laboratory

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Factor Analysis in Nuclear Medicine - Useful but Dangerous

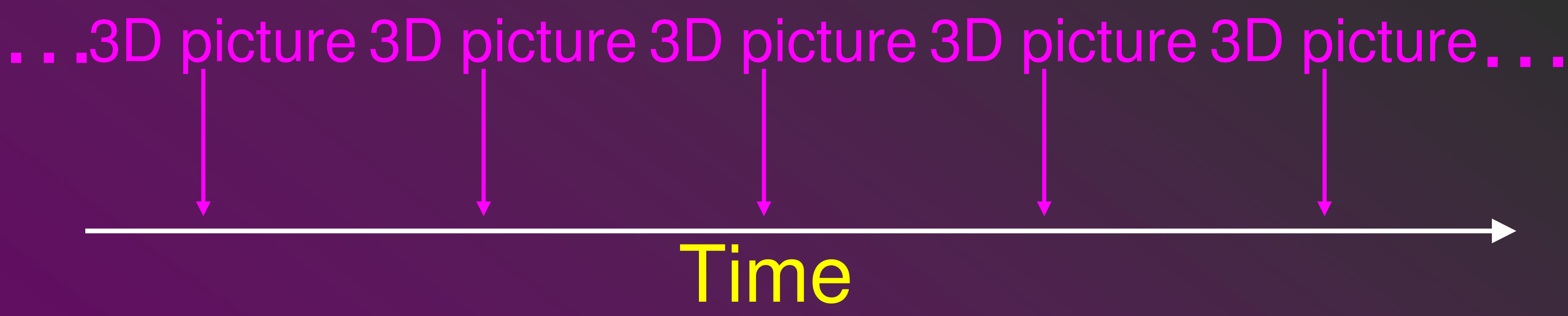
- Introduction
  - What is the factor analysis and where can it be applied
- Part I
  - Why dangerous
- Part II
  - Why useful

Goal

- **Familiarize**
- **Caution**
- **Interest**

Dynamic Imaging

- 4 Dimensions



Dynamic Imaging

- Series of 3D-images each taken at a different time
- Differential uptake of pharmaceutical can be obtained

Factor Analysis in Dynamic Imaging

- Series of 3D-images each taken at a different time

A method of obtaining of the differential time uptake for a given organ or a region in 3D image

- Differential time uptake of pharmaceutical can be obtained

Examples of Dynamic Imaging

- Planar Single Photon

  ^{99m}Tc-MAG3 for estimating kidney GFR (2Dimensions)
- SPECT

  ^{201}Tl, ^{99m}TC-Teboroxime for heart perfusion
- PET

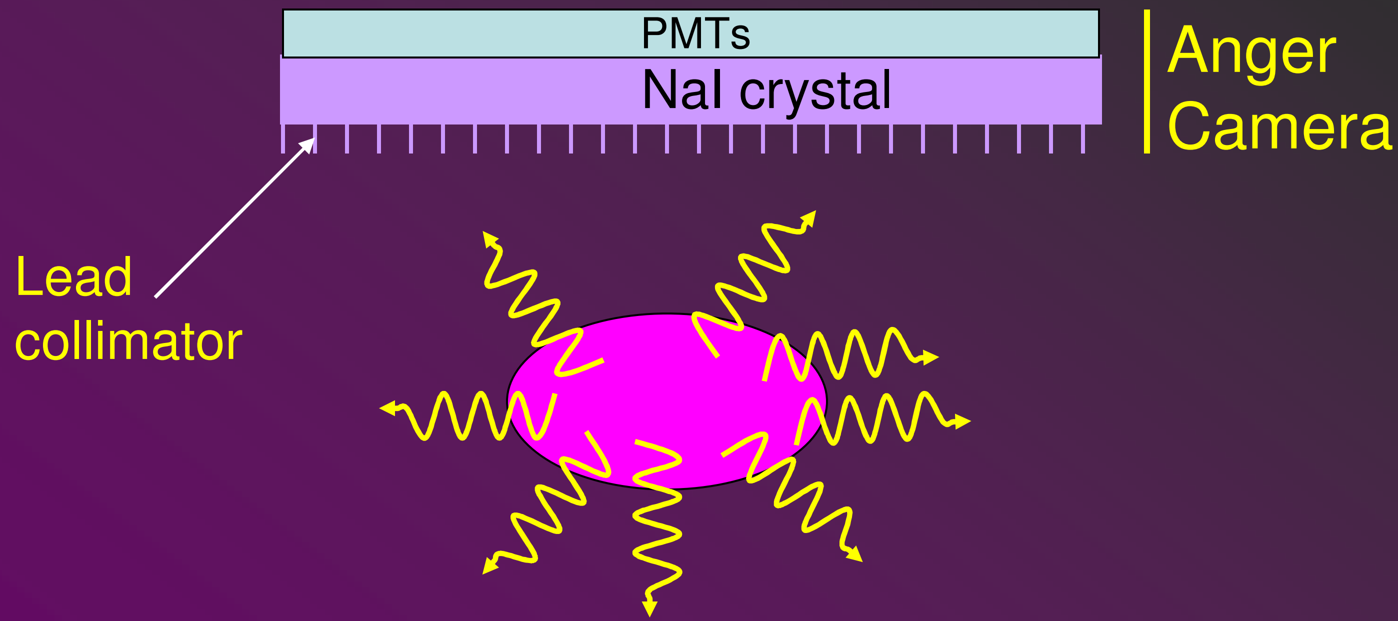
  ^{15}O-Water for brain activation study
- PET

  ^{11}C-Raclopride ^{18}F-DMFP for dopamine studies in a brain
- PET

  ^{13}N-Ammonia, ^{82}Ru,^{18}F-FDG,^{18}F-fatty acids for cardiac studies

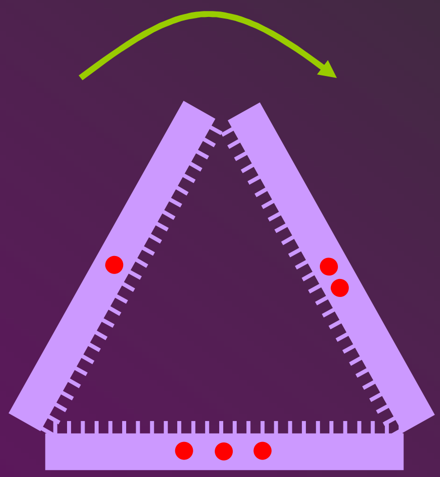
Acquisition of Dynamic Data

- Planar
  - 2D images
  - Stationary detector



Acquisition of Dynamic Data

- FAST SPECT
  - Fast rotation
  - Usually done with 3 detector camera



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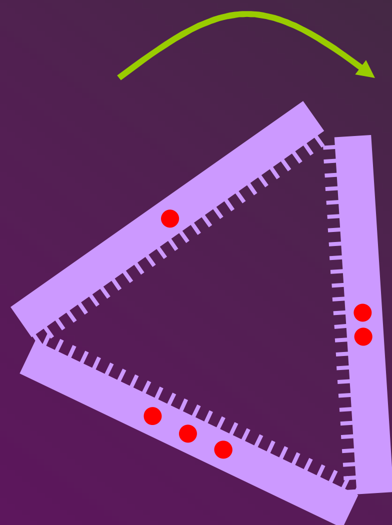

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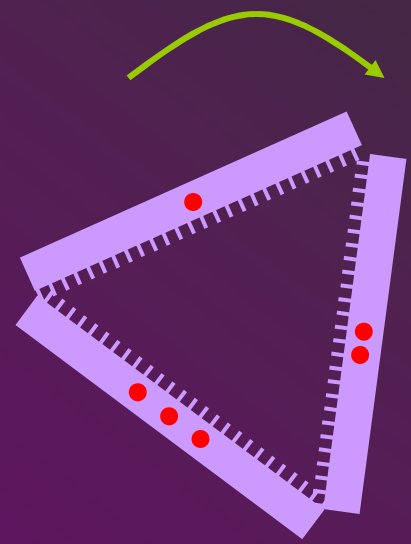

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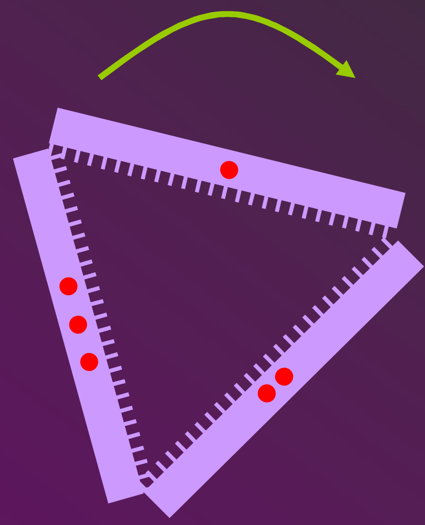

Acquisition of Dynamic Data

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  - Fast rotation
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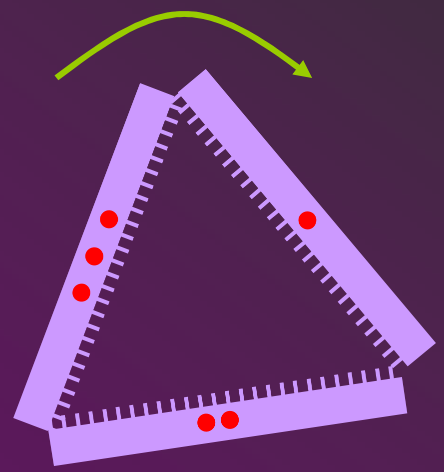

Acquisition of Dynamic Data

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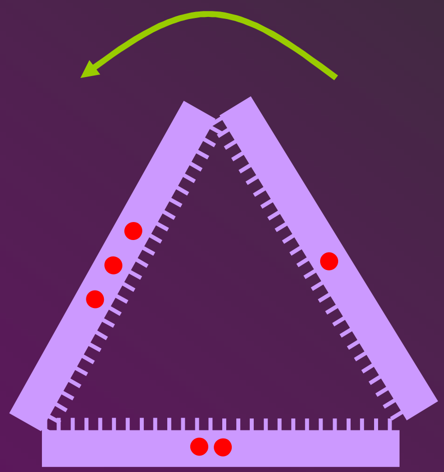

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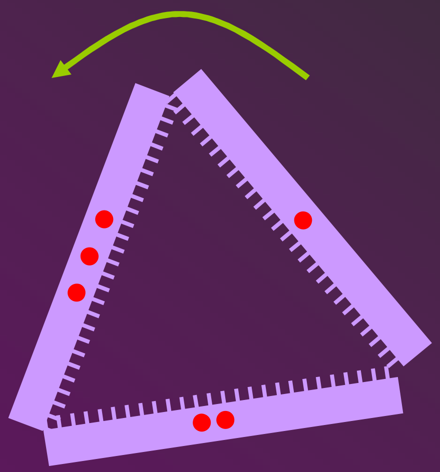

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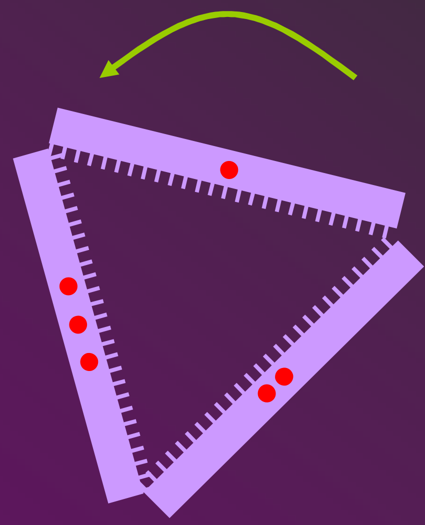

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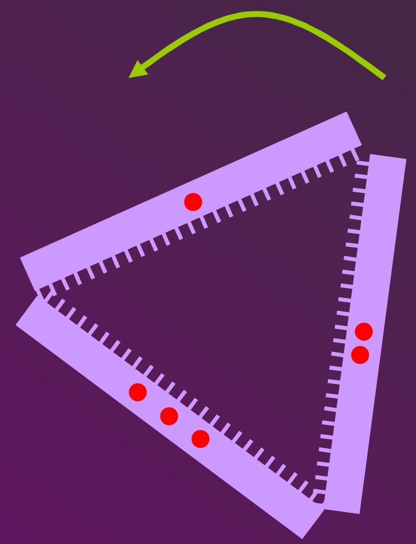

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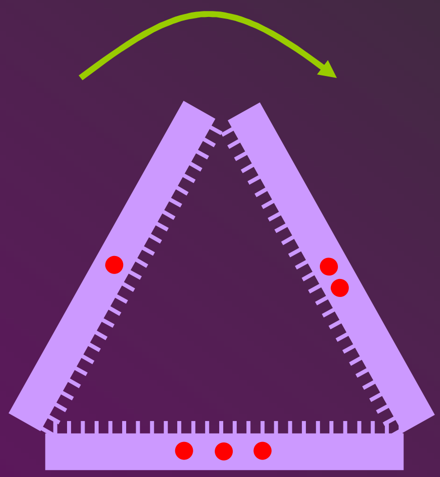

Acquisition of Dynamic Data

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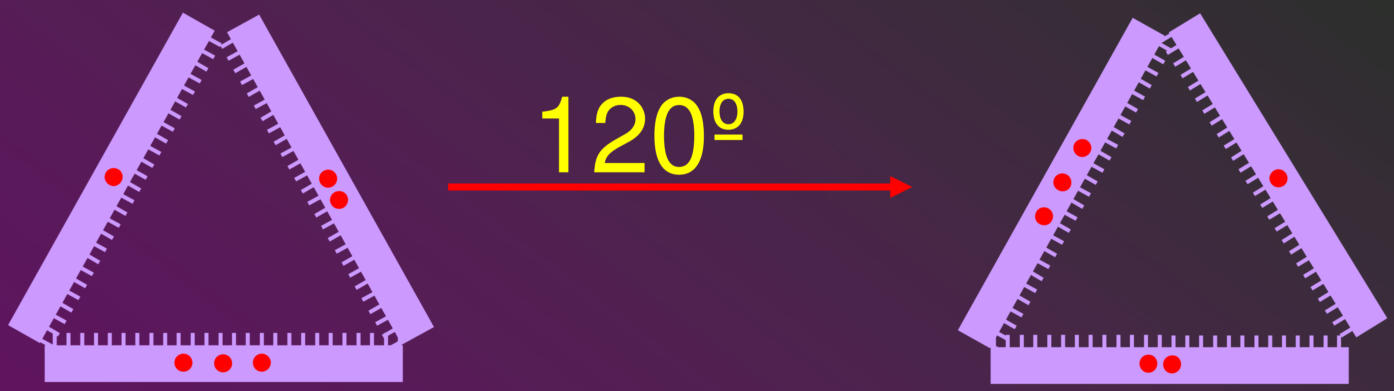
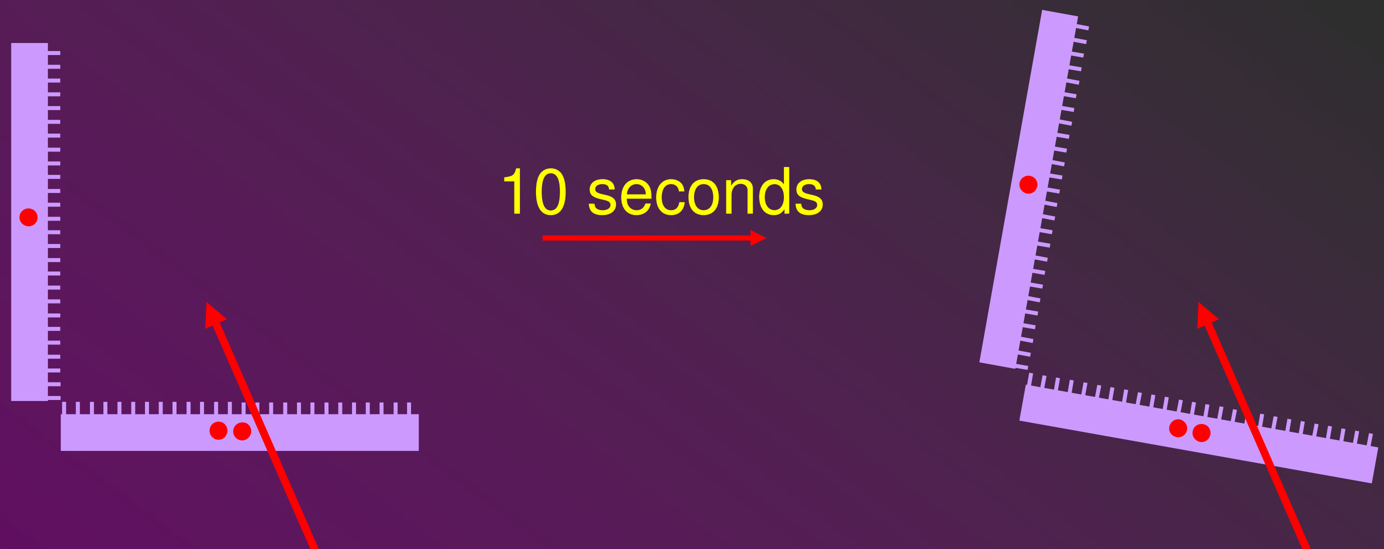


•Each 120º rotation gives one 3D image

•The best time resolution of such system: about 10 seconds

Acquisition of Dynamic Data

- SLOW SPECT
  - Slow rotation
  - Inconsistent projections



•Detectors during this projection see different activity than during this one

•Standard reconstruction of such inconsistent data will **FAIL**

Acquisition of Dynamic Data

- SLOW SPECT
  - Slow rotation
  - Inconsistent projections
  - Special reconstruction techniques have to be used to deal with the inconsistency in the Slow SPECT acquired dynamic data
  - Factor analysis can be used (Sitek *at al* JNM 2001)

Acquisition of Dynamic Data

- PET

Dynamic PET

EASY compared to SPECT

"… and sometimes I dream about inherent dynamic PET acquisitions…"

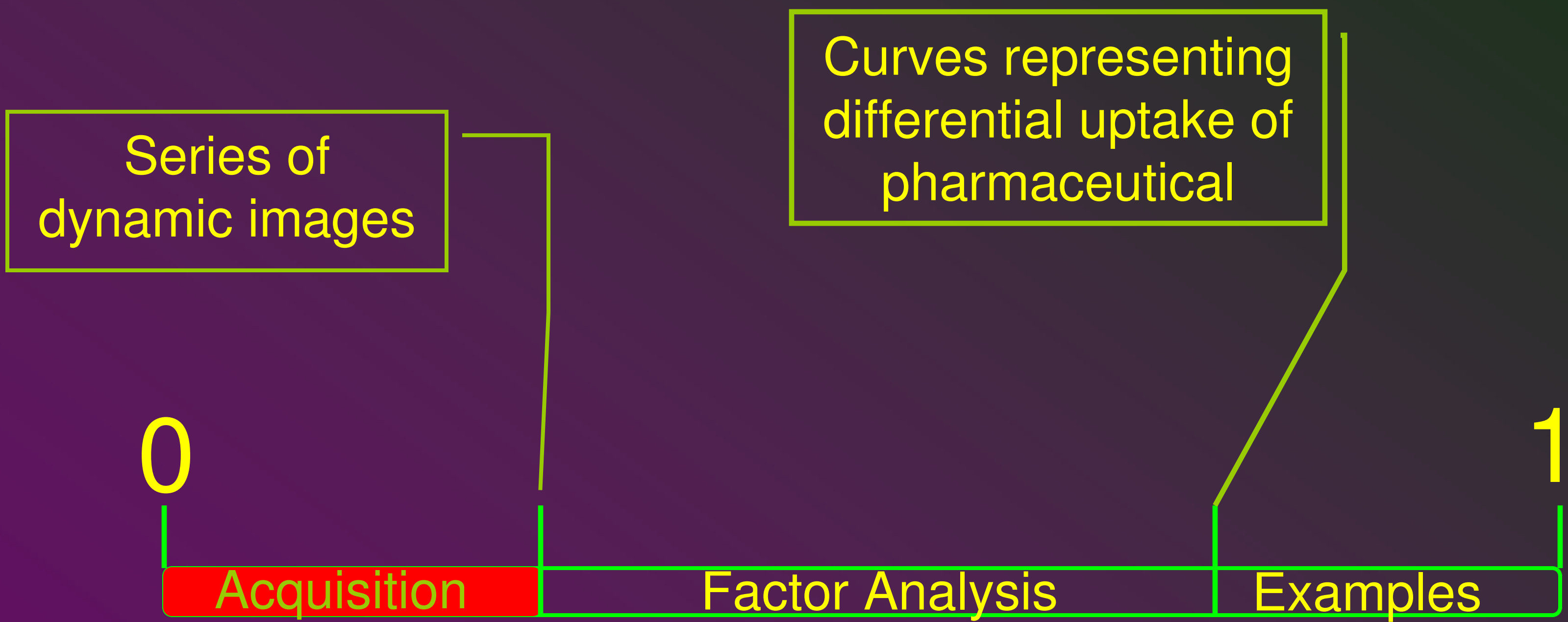
taken from "My Life as a Nuclear Medicine Physicist"

Acquisition of Dynamic Data

- PET

  - Projections are consistent - ALWAYS
  - Theoretically any time resolution is possible

Presentation Gauge*



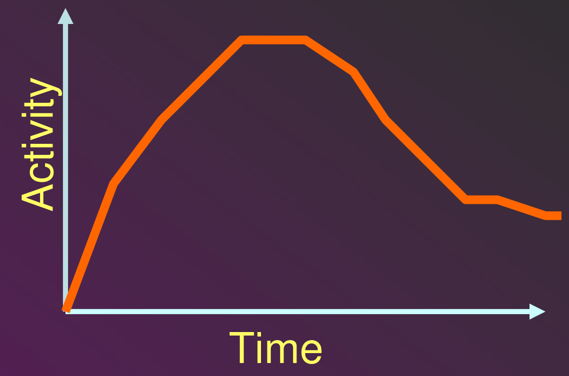
*- any correlation with the time of the presentation should not be assumed

Factor Analysis

- Terminology

  Factor Analysis = Factor Analysis of Dynamic Structures (FADS)

  Time Activity Curve (TAC)



Factor Analysis

- Factor model assumes that in the image there are groups of pixels that have the same time behavior
- Examples: Liver, heart, heart defect *etc*.

Factor Analysis

- Factor Analysis finds groups of pixels that have a similar time uptake, and also determines the corresponding TAC
- Number of groups is predetermined (usually up to 4)

A factor is the TAC that corresponds to a group

A factor coefficient image is a spatial definition of a group

Factor Analysis

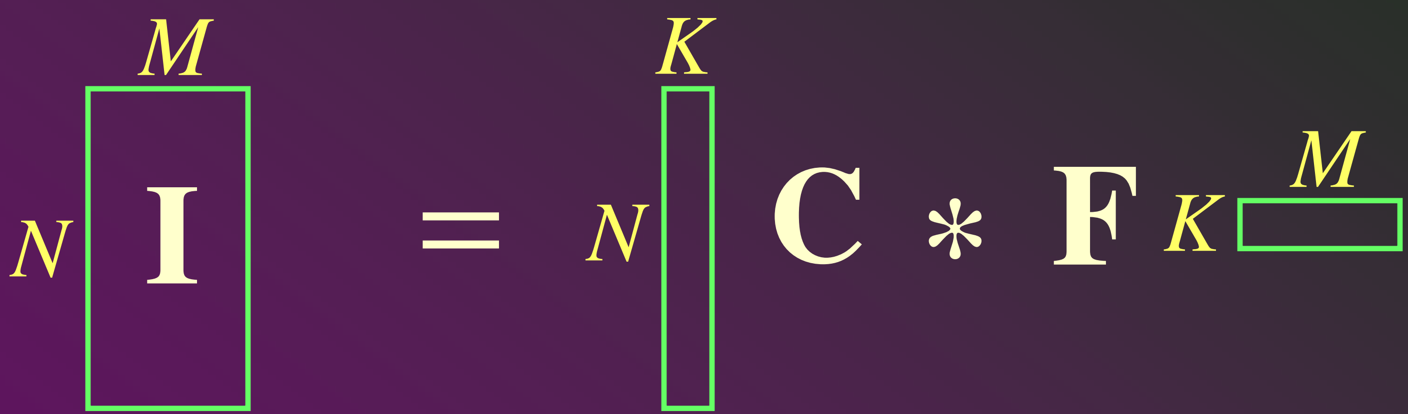
Image from dynamic sequence acquired at time t = Image of factor coefficients for first factor times value of that factor for time t + Image of factor coefficients for second factor times value of that factor for time t +…

$$\mathbf{I}(t) = \mathbf{C}_1 F_1(t) + \mathbf{C}_2 F_2(t) + \ldots$$

$$\mathbf{I} = \mathbf{C} * \mathbf{F}$$

Factor Analysis

Dynamic sequence = Factor coefficients * Factor



N	Number of pixels in each dynamic image (128x128x30)
M	Number of dynamic images (100)
K	Number of factors (4)

Factor Analysis

$$\mathbf{I} = \mathbf{C} * \mathbf{F}$$

The goal of performing FADS is to obtain matrix **C** and matrix **F** having as input matrix **I**

Reminder:

Matrix C corresponds to image of the organs with similar uptake (in cardiac imaging it could be heart tissue, left or right blood pool, liver…)

Matrix F corresponds to time behavior of those regions

Matrix I is the measured dynamic sequence

Factor Analysis

$$\mathbf{I} = \mathbf{C} * \mathbf{F}$$

Is it hard to get C and F using above equation ? **Not really**

Reminder:

Matrix C corresponds to image of the organs with similar uptake (in cardiac imaging it could be heart tissue, left or right blood pool, liver…)

Matrix F corresponds to time behavior of those regions

Matrix I is the measured dynamic sequence

Factor Analysis

$$\mathbf{I} = \mathbf{C} * \mathbf{F}$$

The most popular methods of solving the above is the apex seeking method by *Di Paola et al IEEE TNS* 1980 *+ Variations on a Theme..*

Reminder:

Matrix C corresponds to image of the organs with similar uptake (in cardiac imaging it could be heart tissue, left or right blood pool, liver…)

Matrix F corresponds to time behavior of those regions

Matrix I is the measured dynamic sequence

Factor Analysis

$$\mathbf{I} = \mathbf{C} * \mathbf{F}$$

Dangerous Point: The solution that come out of these (apex seeking) methods is mathematically not unique if only non-negativity constraints are used

Reminder:

Matrix C corresponds to image of the organs with similar uptake (in cardiac imaging it could be heart tissue, left or right blood pool, liver…)

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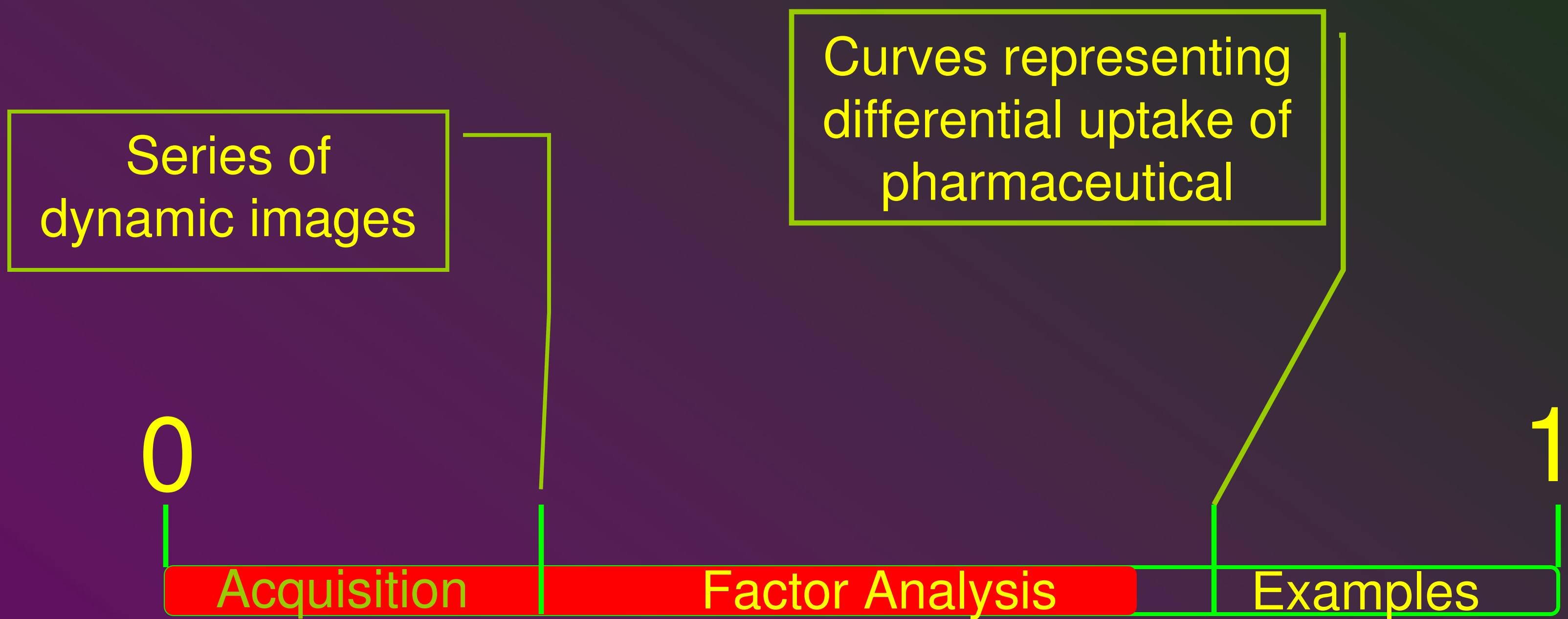
Matrix I is the measured dynamic sequence

Presentation Gauge*



*- any correlation with the time of the presentation should not be assumed

Factor Analysis

Demonstration of the non-uniqueness for 2 factor model. Matrices C_1, C_2, F_1, F_2 are non-negative

$$I = C_1 F_1 + C_2 F_2 \qquad (1)$$

$$I = C_1F_1 + C_2F_2 + aC_1F_2 - aC_1F_2 \qquad (2)$$

$$I = C_1(F_1 - aF_2) + (C_2 + aC_1)F_2 \qquad (3)$$

$$I = C_1F'_1 + C'_2F_2 \qquad (4)$$

Matrices $C_1, C'_2 = C_2 + aC_1, F'_1 = F_1 - aF_2, F_2$ are equally good as long as non-negative

Factor Analysis

$$\mathbf{I} = \mathbf{C} * \mathbf{F}$$

In all examples that will be presented factor analysis based on least squares (Sitek *at al PMB* 2000, *IEEE TMI* 2002) will be used for solving the above

Reminder:

Matrix C corresponds to image of the organs with similar uptake (in cardiac imaging it could be heart tissue, left or right blood pool, liver…)

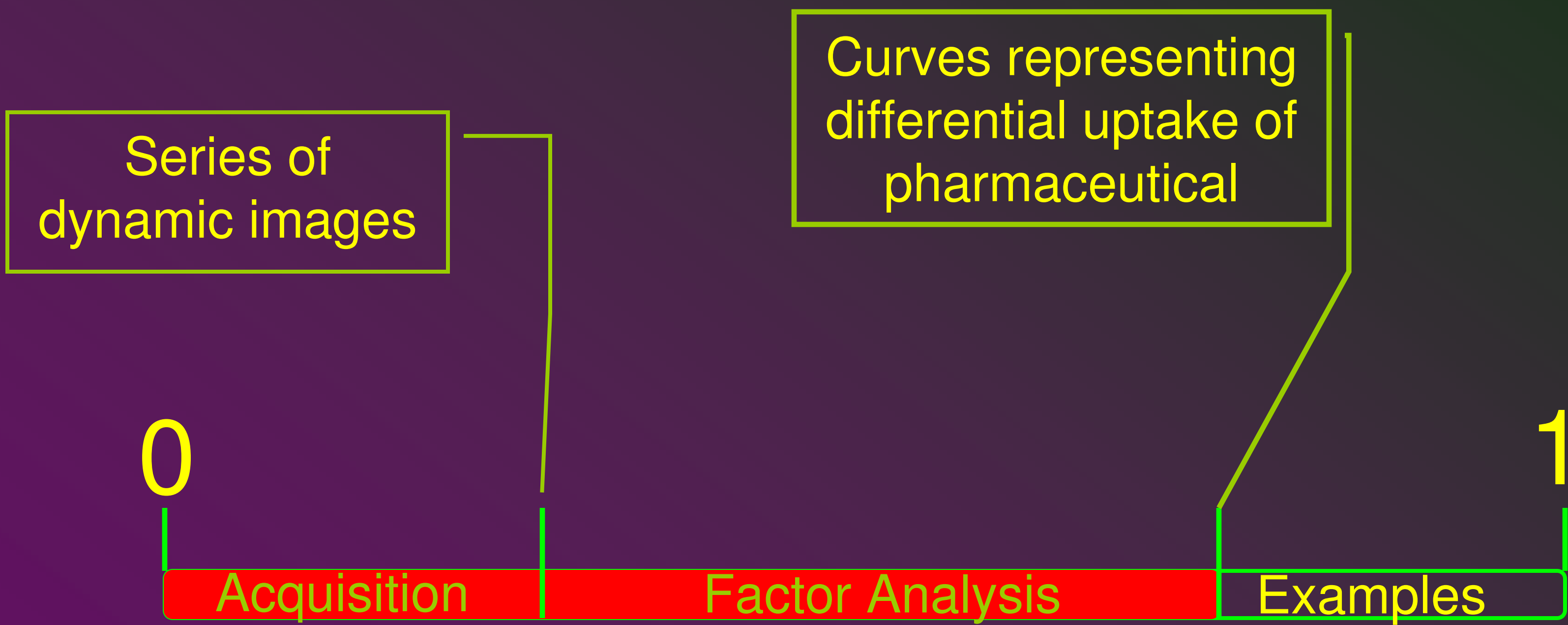
Matrix F corresponds to time behavior of those regions

Matrix I is the measured dynamic sequence

Factor Analysis

- **LS-FADS** method minimizes the difference between the model and the data with non-negativity constraints (results similar to the apex-seeking)
- **PLS-FADS** method minimizes the difference between the model and the data with the non-negativity constraints and with the correction for non-uniqueness

Presentation Gauge*



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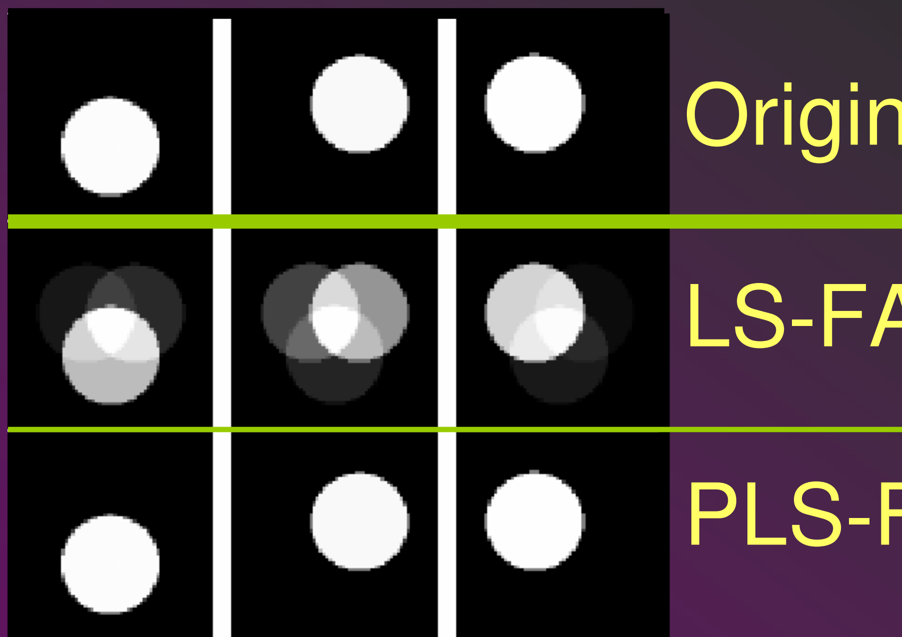
FADS examples

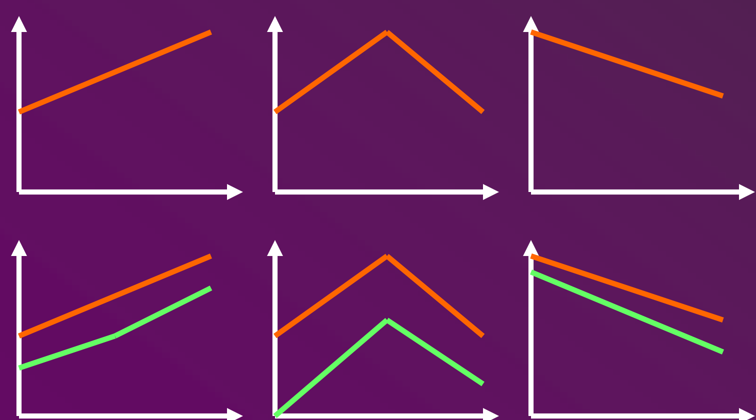


Original

LS-FADS

PLS-FADS

Computer Simulation



Original or PLS-FADS

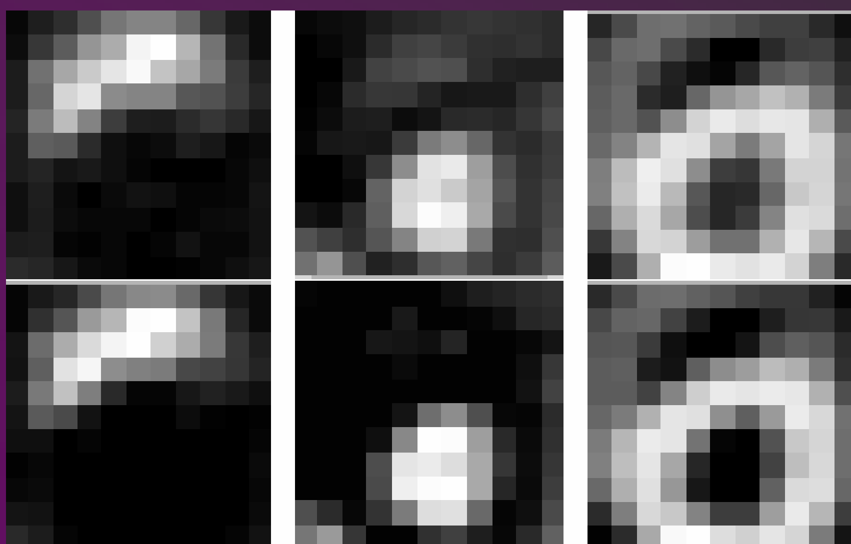
LS-FADS

FADS examples

Cardiac canine

^{99m}Tc-Teboroxime

RV LV Tissue



LS-FADS

PLS-FADS

IMAGING PROTOCOL

FAST SPECT 179 image dynamic series. Images taken every 7 seconds at rest and stress

Factor Analysis

Demonstration of the non-uniqueness for 2 factor model. Matrices C_1, C_2, F_1, F_2 are non-negative

$$I = C_1 F_1 + C_2 F_2$$

$$I = C_1F_1 + C_2F_2 + aC_1F_2 - aC_1F_2$$

$$I = C_1(F_1 - aF_2) + (C_2 + aC_1)F_2$$

$$I = C_1F'_1 + C'_2F_2$$

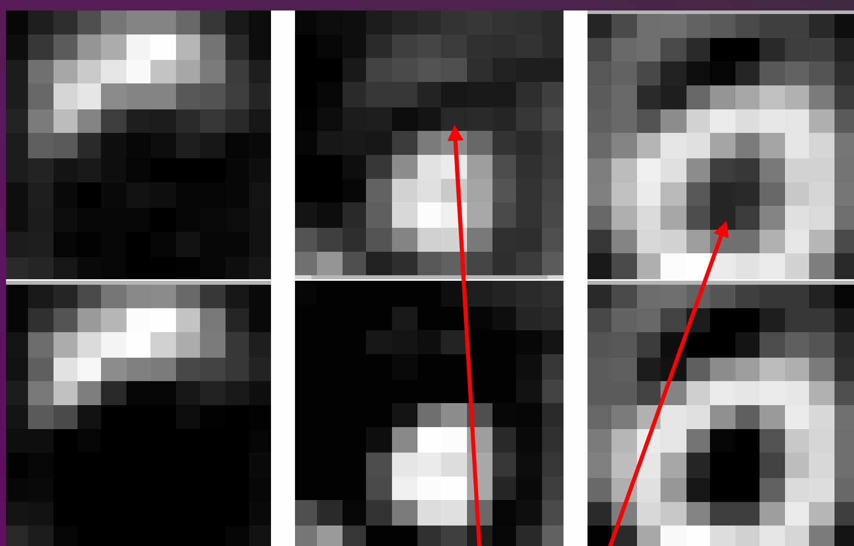
Matrices $C_1, C'_2 = C_2 + aC_1, F'_1 = F_1 - aF_2, F_2$ are equally good as long as non-negative

FADS examples

Cardiac canine

^{99m}Tc-Teboroxime

RV LV Tissue

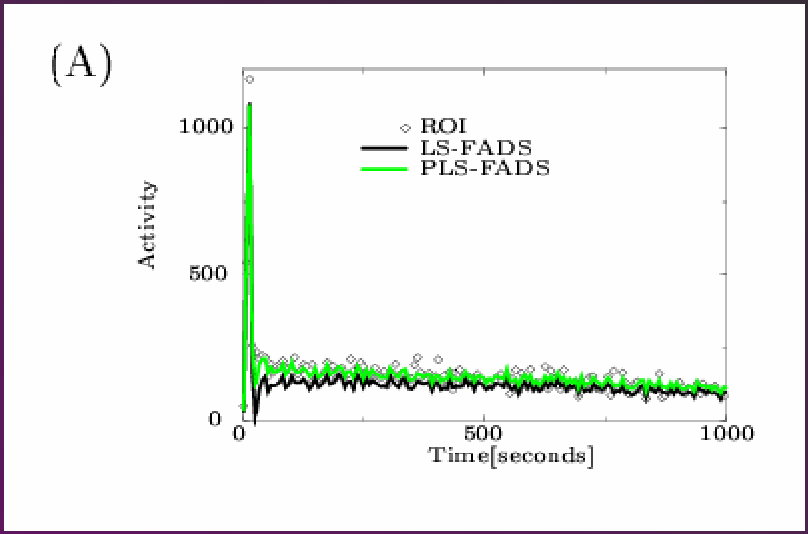


LS-FADS

PLS-FADS

Decreased contrast

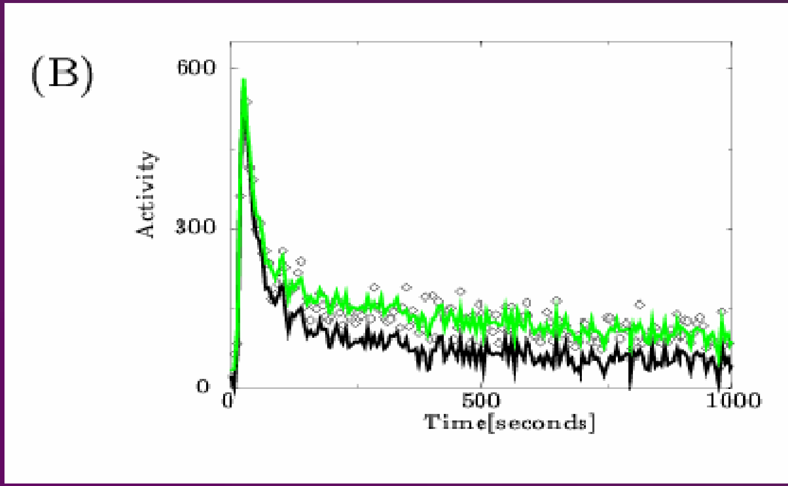
FADS examples

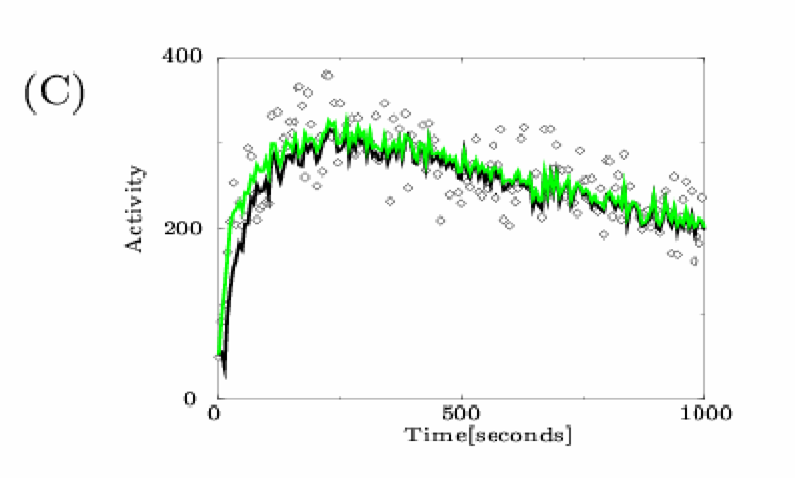


RV

Cardiac canine

^{99m}Tc-Teboroxime





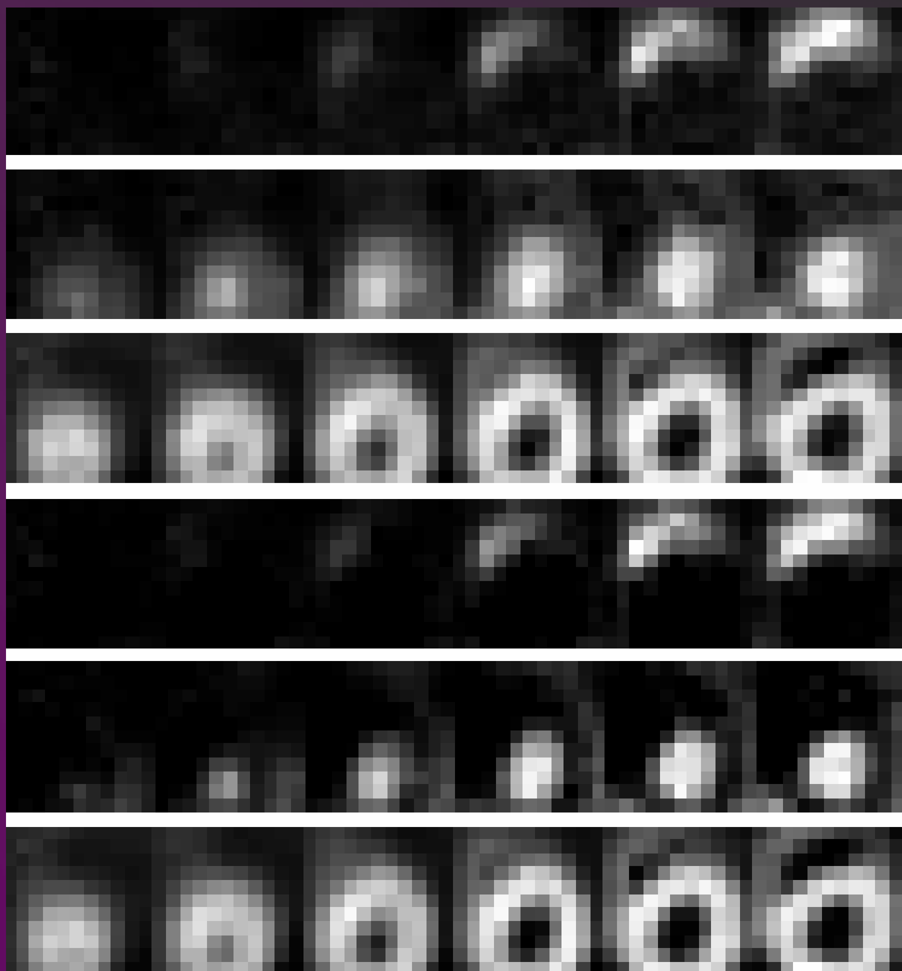
LV

Tissue

FADS examples

Cardiac canine

^{99m}Tc-Teboroxime



RV

LV

Tissue

RV

LV

Tissue

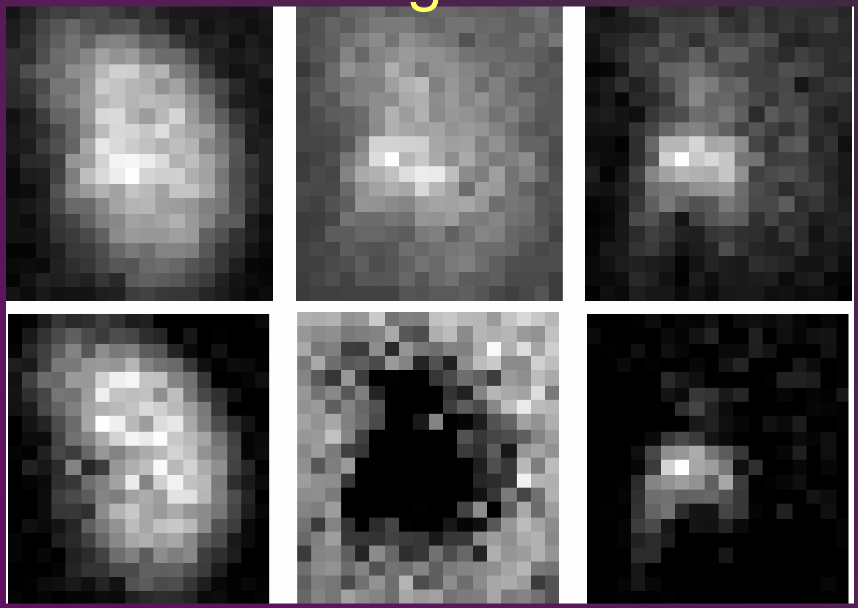
3D

FADS examples

Kidney

^{99m}Tc-MAG3

Cortex Background Ureter



LS-FADS

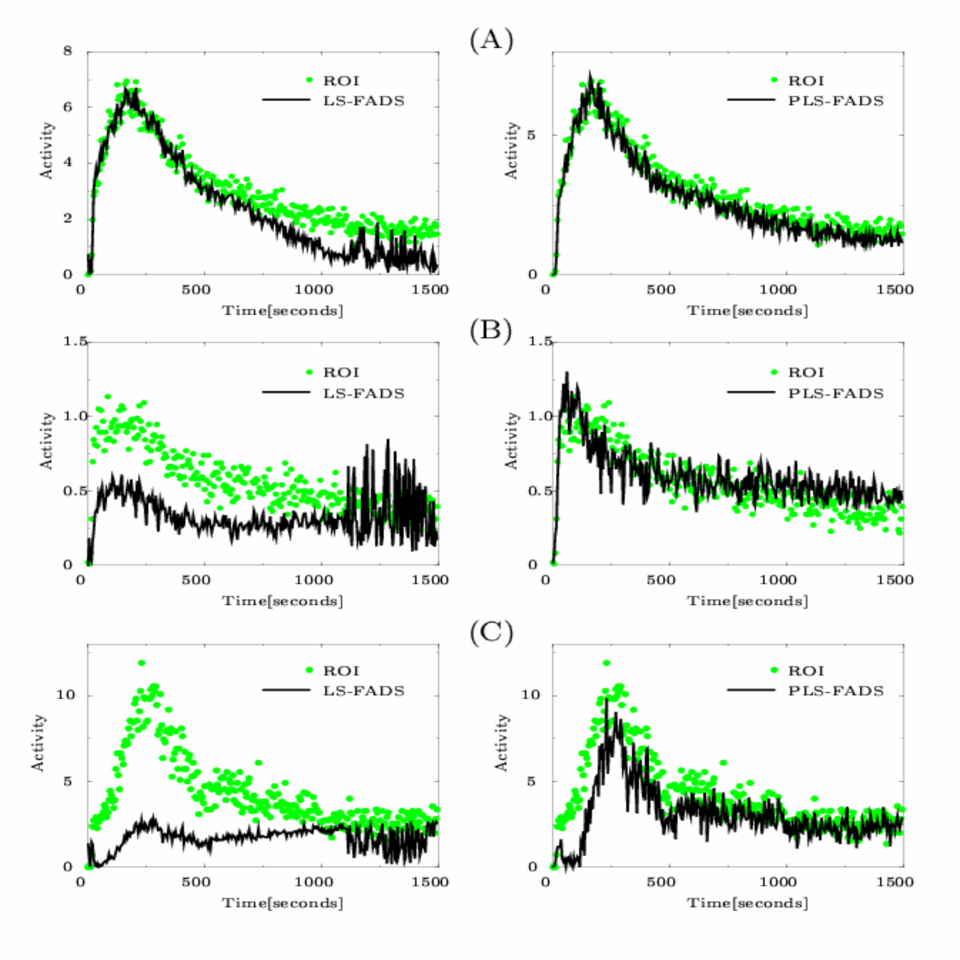
PLS-FADS

IMAGING PROTOCOL

Planar - 300 image dynamic series.
Images taken every 5 seconds.

FADS examples

LS-FADS PLS-FADS



Cortex

Background

Ureter

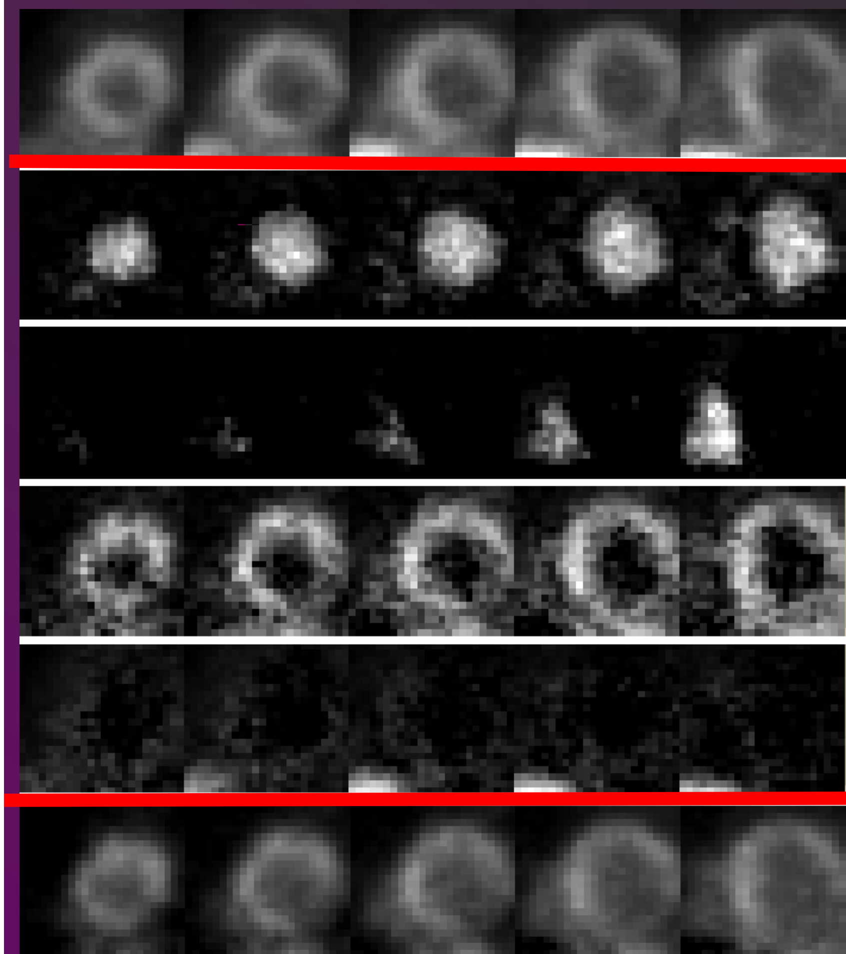
Patient kidney

^{99m}Tc-MAG3

FADS examples

- With no non-uniqueness correction – 2-3 factors
- If the non-uniqueness is properly addressed even 4 factor-FADS works ! Wow !

FADS examples



Patient cardiac

^{99m}Tc-Teboroxime

IMAGING PROTOCOL

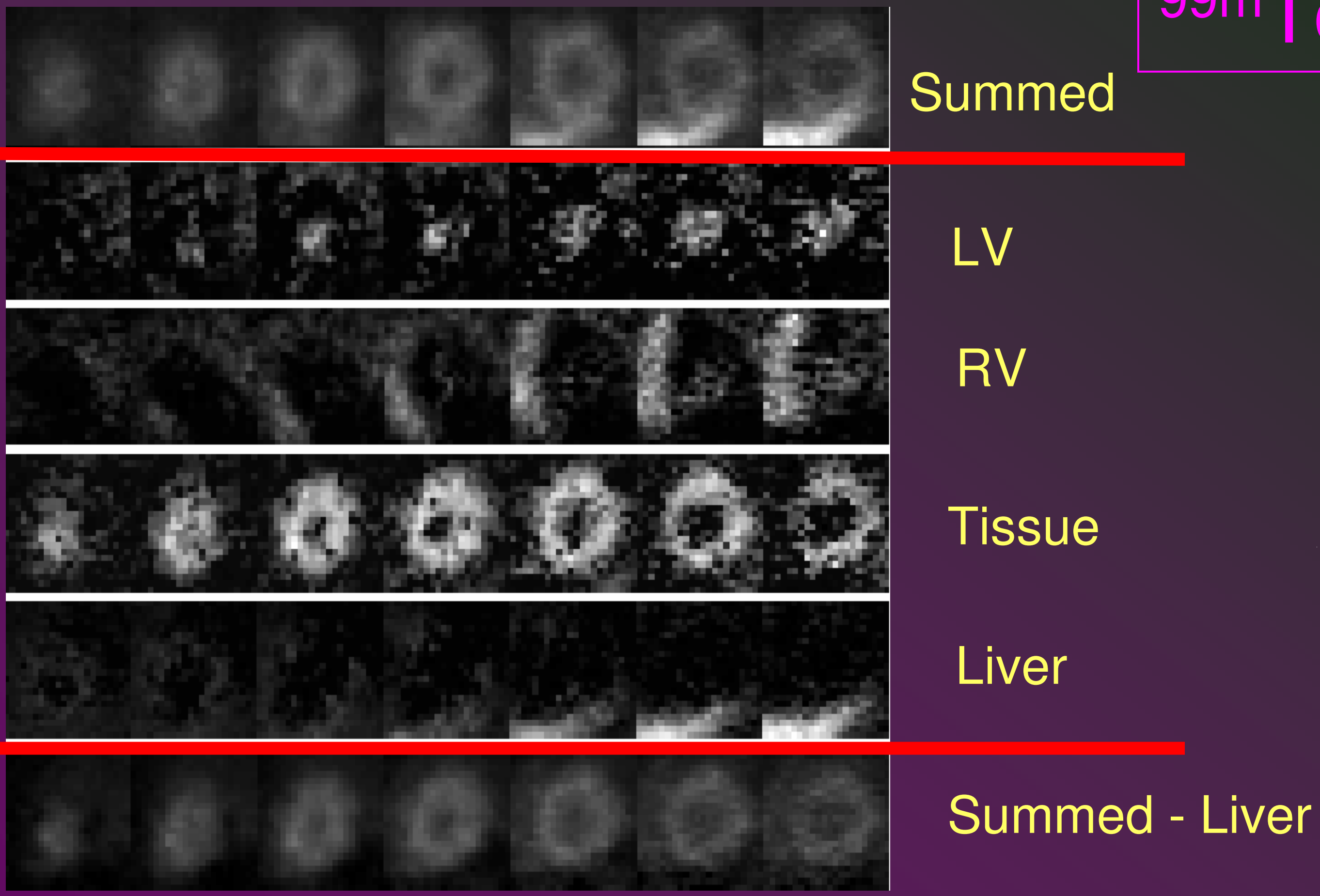
FAST SPECT - 90 image dynamic series. Images taken every 11 seconds in rest and stress

PLS-FADS

FADS examples

Patient cardiac

^{99m}Tc-Teboroxime



PLS-FADS

FADS examples

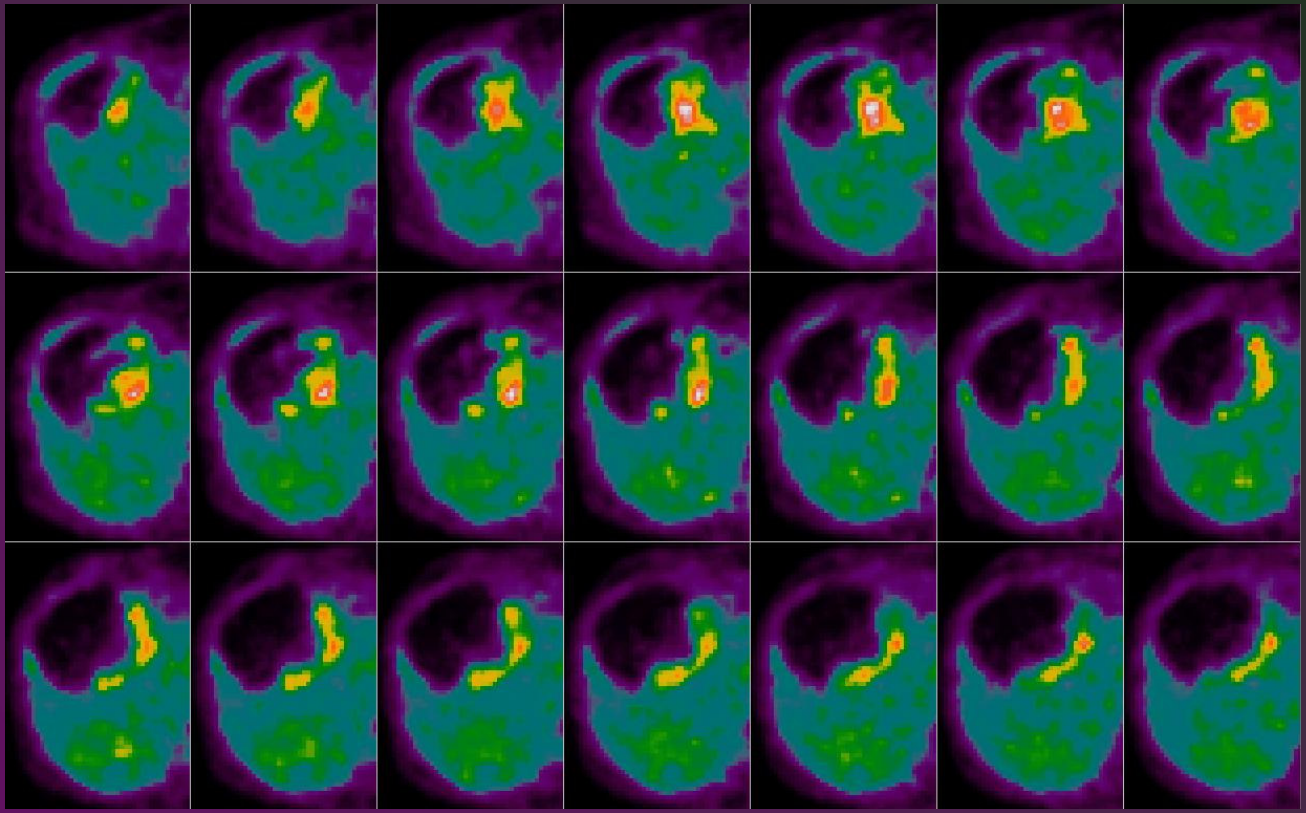
IMAGING PROTOCOL

PET - 30 image dynamic series
Images taken over variable periods.

Patient liver

^{18}F-FDG

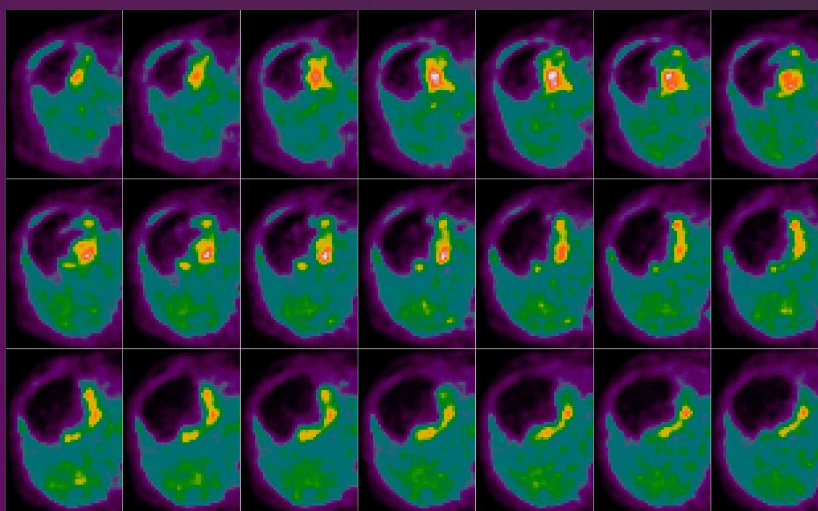
21 slices of summed image (right lobe of the liver)



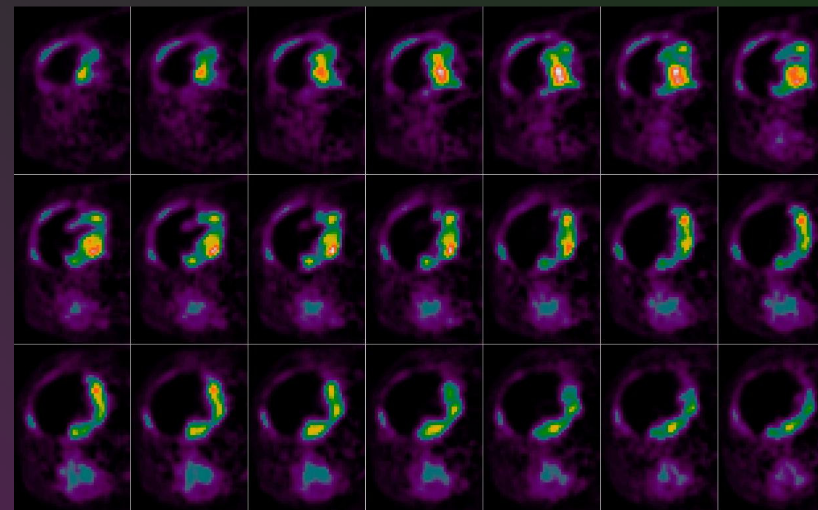
FADS examples

Patient liver ^{18}F-FDG

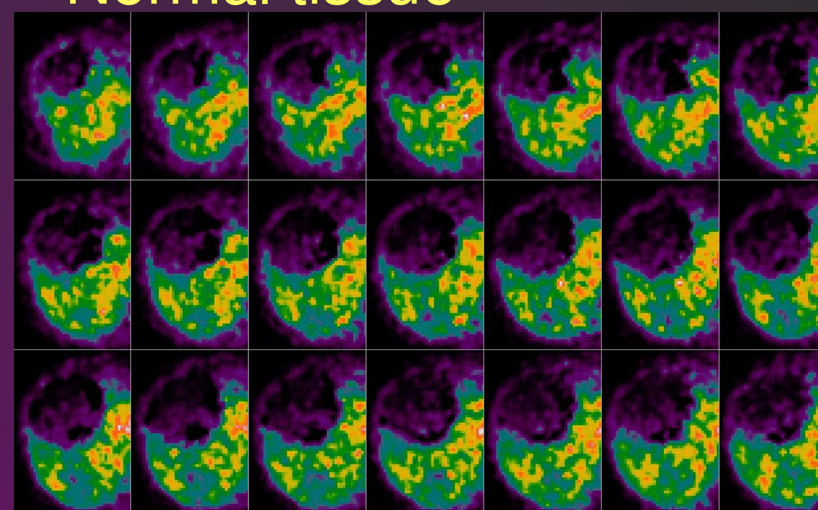
Summed



Tumor



Normal tissue



Summary

- Is this non-uniqueness a problem?
- Does that mean that the factor analysis used in the past 20 years is all wrong?
- Can the non-uniqueness be corrected?

YES

YES NO MAYBE

YES

Summary

•Does that mean that the factor analysis with non-negativity constraints used in the past 20 years is all wrong?

YES

These factors create horrible non-uniqueness problems

MAYBE

These factors create some or none non-uniqueness problems

NO

These actually give UNIQUE answer

Summary

- Is this non-uniqueness a problem?
- Does that mean that the factor analysis used in the past 20 years is all wrong?
- Can the non-uniqueness be corrected?

YES

YES NO MAYBE

YES

Summary

- Part I
  - Why dangerous

Non-uniqueness

- Part II
  - Why useful

Summary

- Excellent for image segmentation and separation of the overlapped regions
- Very good for extraction of TACs (better than ROI measurements)
- Semi-automatic

"… and they are all this only if you know how to avoid their dangers"

taken from "My Life as a Nuclear Medicine Physicist"

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Thank you